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A
TREATISE
ON
BRACING
WITH ITS APPLICATION TO
BRIDGES

AND OTHER STRUCTURES OF WOOD OR IRON.

BY
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WITH 156 ILLUSTRATIONS ON STONE.



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P R E F A C E.

It is now several years since the description of bracing chiefly dwelt upon in the following pages suggested itself to the author; and he was surprised to find that a method of such simplicity and evident excellence should have been employed in only a few unimportant instances, and, in the majority of these, in a mixed or not very evident way.* This naturally led him to investigate its qualifications, and it was his expectation to have had opportunities, in the exercise of his profession, of making practical use and exemplification of the results.

*The first example that is likely to occur to the reader is that of the spandrils of Southwark Bridge; but the *arch*, from its construction and *depth* of material, is quite independent of additional bracing, and the *use* of the *lozenges* of the spandrils is merely to connect the arch with the roadway; thus, the Sunderland Bridge, which is of nearly the same span and of greater rise, and composed of voussoirs of less depth and inferior character for rigidity, is, nevertheless, without spandril-bracing.

A design somewhat similar to the above, represented by Figure 19, is sometimes used for engine-shafts, etc., and its bracing power is, in such cases, brought into action.

Another, and, though insignificant, seemingly a very clear application of the method is that employed to brace the transverse pipes between the main chains of the Menai Bridge.

But the most decided case of its employment, in its simplest form, which the author has met with is that figured in the March part of "The Builder" of the year 1850 (page 100, Vol. VIII.); it is the wrought-iron roof over the Strasburg Railway Station at Paris.

Town's Lattice Bridge depends, no doubt, for its strength upon the method, but presents an example of it in a very compound state, and which might have been adopted without a knowledge of the efficiency of the method in its simplest form. (Figure 22.)

Such opportunities, however, not having as yet occurred, he adopts the less congenial means of the pen to place his investigations in such a position that they may be capable of becoming useful. In these investigations the simpler geometrical processes have been preferred to those of a more abstruse character.

The first draft of the work was strictly confined to the particular kind of bracing above referred to, which may be appropriately named the *double-acting* or *triangular* method ; but, upon after-consideration, the plan has been somewhat extended, as it was judged that a few moderate additions would render the work more useful, and better fitted, though very imperfectly, to supply an important desideratum in the library of the engineer.

For dignifying the little work with the title of “A Treatise on Bracing,” it is offered as an explanation that, though devoted almost exclusively to one method, the results obtained may be easily applied to all other varieties, with the exception of *plate-bracing*. This last kind, indeed, is of so peculiar a nature that a completely different process must be employed to elucidate its action ; probably the results of experiment must be principally depended upon to give us correct ideas of its capabilities.

As *bridges* are the structures in which bracing is most extensively employed, much will be said of them ; but, as the intention is merely to explain its application, those parts which either influence or are influenced by the character of the bracing alone come strictly within the scope of this treatise. In a few instances the author may have gone beyond this intention, in the belief that the space so employed would not be deemed by the reader unprofitable.

In conclusion, the writer begs to say that in launching his little bark upon the seas of publicity, he is not expectant of its passing triumphantly through the testing billows of criticism ; these may disclose many deficiencies in its structure—with some of which, however, its architect is already acquainted ; but the first vessel built upon a new principle is always, in some degree, experimental, aiding man to construct in the future a perfect one of the same class. Seldom, upon a new subject, was a volume ever published that had not to undergo many changes and enlargements ere it ranked as a standard.





CONTENTS.

CHAPTER INTRODUCTORY ON BRACING.

DEFINITIONS—ELEMENTARY BRACED FORMS—FAULTY STRUCTURES, 1

PART FIRST—THEORY AND DATA.

CHAPTER I.

ON THE DOUBLE-ACTING OR TRIANGULAR METHOD OF BRACING.—Development—Efficiency—Advantages, 5

CHAPTER II.

APPLICATION OF THE TRIANGULAR METHOD TO VARIOUS STRUCTURES.—New View of the Action of a Bracing—Division of Structures into Four Classes—Table of Symbols Used, 7

CHAPTER III.

THE PRESSURES THAT MAY ACT IN STRUCTURES OF CLASS FIRST.—Process of Investigation Adopted—Examples—Formulæ for the Maximum Strains in the Braces and in the Main Beams, 10

CHAPTER IV.

THE PRESSURES THAT MAY ACT IN CLASS SECOND.—Strains in the Braces, etc.—Transition into Class Third—Triangular Forms, 14

CHAPTER V.

THE PRESSURES THAT MAY ACT IN CLASS THIRD.—Proper Form of Arch—Principle of Investigation—Examples—Means of Facilitating its Application—General Formulæ—Table of Formulæ for the Strengths of the Braces—Pressures in the Arch and Tie—A Form having More Uniform Strains—Effects from the Line of Pressures not lying in the Arch—The Arch with External Bracing, 18

CHAPTER VI.

THE PRESSURES THAT MAY ACT IN CLASS FOURTH, 26

CHAPTER VII.

ON THE INCLINATION OF THE BRACES, ETC.—The Proper Degree of Inclination of the Braces—Poly-systemed Bracings—Plate-Bracing, its varieties very numerous—Comparison of the Methods, etc., . 27

PART SECOND—CONSTRUCTION.

CHAPTER I.

VARIOUS BRACINGS REQUIRED IN DIFFERENT STRUCTURES, 31

CHAPTER II.

CONSTRUCTION AND APPLICATION OF THE BRACES.—General Remarks—Wooden Braces—Ditto with Iron Sockets—Iron Braces, Cast and Wrought, 35

CHAPTER III.

CONSTRUCTION OF OTHER PARTS CONNECTED WITH THE BRACING.—Laminated Wooden Arches and Beams—Springings of Wooden Arches—Iron Structures—Springings of Iron Arches, 39

CHAPTER IV.

SUPPLEMENTARY.—The First Class, though Weaker, has many Advantages over the Others—Triangular Tube Bridge—Internal generally Superior to External Bracing for an Arch—Strength of Wooden Braces—The whole Length of a Structure must be Braced—Designs Compounded of the First and Third Classes—The Strains in the Longitudinals of Class First may be spread more uniformly over them—Further observations on the relation between the Arch and the Line of Pressures—The Line of Tensions in the Inverted Arch, 41



TREATISE ON BRACING.

INTRODUCTORY CHAPTER.

DEFINITIONS.

It is necessary to state here the sense in which several terms will be employed; as it has been found expedient to restrict the meaning in most cases, thus rendering the words so much the more precise.

When a construction is so framed that, under the action of any arrangement of forces, the angles cannot alter, nor therefore the shape, it is described as a *completely braced*, or simply a *braced*, structure.

When a fabric is adapted to retain its shape under the action of a force applied in one direction only, it is said to be only partially braced, or braced sufficiently for the circumstances; as in the case of the simple roof, Figure 2.

When the external figure of the structure is not, of itself, such as to ensure the retention of its form (*i.e.*, when not a triangle), the addition to effect this is called the *bracing*, and each piece is named a *strut*, a *tie*, or a *brace*, according to the office it performs.

When a part is only required and adapted to resist compression, it is called a **STRUT**,* and in the diagrams of this work, when it can conveniently be done, will be represented by two lines, as shown by *a*, Figure 1.

When a part is only required and adapted to act against a

* In speaking of the parts of a structure, if it be desired to distinguish those struts or ties used for bracing it from others, they may be designated as *bracing struts* or *bracing ties*.

force of tension, it is called a tie-beam, tie-piece, tie-bar, or simply a TIE, and will be represented by a single strong line, as *b*.

When a part is required and adapted to act both as a strut and a tie, it is called a double-acting piece, complete brace, or simply a BRACE, and will be represented by two lines, as for a strut, with the addition of a central stronger one, as for a tie; this is shown by *c*.

When the parts of a *braced* figure are so long as to be in danger of bending, and this is prevented by certain additions, these are called, collectively, the *trussing*, and the beams so stiffened are said to be *trussed*; the whole figure so treated, when it does not require *bracing*, is called a *truss*. Thus, Figure 2 represents a simple roof-truss; the additional parts, *cf*, *fd*, *fb*, constitute the trussing, and they truss or stiffen the pieces *ae*, *ec*, and *ac*, respectively.

The term *pressure*, consistently with the practice of eminent writers, is used to signify *statical* force, either of a compressive or tensive character.

BRACED FORMS.

As a foundation from which to commence, we assume the following propositions:

PROP. I. In a triangle, an angle cannot increase or diminish without the opposite side also increasing or diminishing.





PROP. II. When the angles of a figure are unchangeable, the shape is unchangeable, and, therefore, the figure is completely braced.

The converse of each of these is also true.

A *triangular* structure (Figure 3), having sides that are unchangeable (by I. and II.), is a completely braced form.

If a quadrilateral figure (Figure 4) alter its shape (by neg. conv. of II.), the angles alter, and, as the sum of the angles must be equal to four right angles, they cannot all increase or all diminish, therefore (considering the diagonals as third sides

of triangles, by 1.) the diagonals must one increase and the other decrease. And hence—

In order that any change of form may take place in a quad- rilateral figure,	{	A diagonal must diminish:—		5	Consequently, all these forms are completely braced, as each does not permit of an effect taking place which would neces- sarily result from a change of figure.	
		A diagonal must increase:—		6		
		Any diagonal must either increase or diminish— both must change:—		7		
						

The above forms are the elements of all thoroughly-braced structures. They are of an insulated character. But there are certain modifications of them which depend upon external resistances; these may be either partially or completely braced, according as their parts are capable of acting in one or two capacities. The Figures 8–11 represent some of these; Figure 11 is an extreme of Figure 8, one of the legs becoming horizontal—it cannot transmit any vertical pressure to its abutment.

There are several forms, frequently employed, which are not sufficiently braced, though very considerable stiffness may be produced in them, but only by the use of excessively thick parts. To save time in describing these and their defects, we have given three series of diagrams in Figures 12, 13, and 14. The first of each, marked A, is a sketch of the objectionable structure; the second, B, represents, in an exaggerated manner, the effects resulting from a weight placed eccentrically; and the remaining diagrams illustrate methods of remedying the defects.

The last series, Figure 14, requires particular consideration. The first modification of it, C, is too often employed as the bracing of bridges, etc. When the weight is placed eccentrically, it is supported on the principle of Figure 11. It will be readily seen, therefore, that when the bridge is long in proportion to its depth, and nearly one-half of its length, $b e$, is loaded, a great force will be brought against the point a , and a very moderate motion of a , or shortening of $c a$, will produce

a considerable depression of the portion at *d*. Hence this cannot be admitted, as a sufficiently braced form, for any but short spans in conjunction with considerable depth of framing. The second modification, D, on the contrary, is an excellent form; in it the principle is quite different; for whereas in the form *c* an eccentric weight is supported wholly (theoretically speaking) by one abutment, in this each abutment contributes support in a degree inversely proportional to the distance of the weight from it. Of course, any of the completely efficient bracings, Figures 5, 6, and 7, for a four-sided compartment, may be introduced. The design E is suited to obviate the particular change shown in B, but it is not braced; other contractions may take place, from the longitudinals approaching nearer to one another.

Part the First.

THEORY AND DATA.

CHAPTER I.

ON THE DOUBLE-ACTING OR TRIANGULAR METHOD OF BRACING.*

To develop this method, let us take the comprehensive case of two parallel beams requiring such bracing between them as shall prevent any change of form in the structure that might result from the action of any forces whatever thereon.

To satisfy this requisite of permanency of shape, the most efficient methods at present in general use, are, in carpentry, those shown by Figures 15 and 16 ; and, in iron-work, those of Figures 17 and 18. Figure 19 gives the best design for cast-iron beams, etc., when of one piece.

These are, no doubt, sufficiently strong and effective as bracings ; but there is, Figure 19 excepted, far more both of material and workmanship used than is absolutely necessary, not only causing needless expense directly, but often also indirectly, as the excess of weight above what is imperatively required may necessitate the provision of a great deal of additional strength in the structure, simply to bear it ; and this, in the case of bridges, would have the effect of considerably diminishing the length of span attainable.

Let us now consider the method of bracing shown by

* This method is founded on the simple braced element shown by Fig. 7. But the insertion of the chapter is warranted by the further elucidation it affords of this most important subject.

Figure 15. Here, as the forces may vary in their direction, all the diagonals require to be strutted; but suppose that the ends of one of these diagonal struts (say $a b$) are so jointed that the strut may be enabled to act also as a tie-bar; such being the case, the other strut ($c d$) of the parallelogram ($a c b d$) is no longer required, for when it would have been of use, the modified one ($a b$), by acting as a tie, is now a sufficient substitute. Giving, then, to one strut of each parallelogram the power to act either as strut or tie, and doing away with the remaining one, the braced structure becomes what is seen in Figure 20 or 21, according to the order of procedure. In the figures, the upright pieces are also represented as fitted with the double-acting joints, so that the iron tie-bars or straps, seen in Figure 15, would be superfluous. In Figure 20, the upright pieces are unnecessary in the bracing, as the point a is fixed in position by the braces $a b$ and $a e$; doing away, therefore, with the upright pieces of Figure 20, our braced framing becomes what is shown by Figure 22, which is the method of bracing advocated; and Figure 21 is evidently a variety of it. Figure 22 might have been deduced from Figure 21 by merely altering the inclinations of the braces, producing an arrangement that would, in most situations, be more efficient.

Figure 22, then, is an effective bracing, capable of resisting a change of form under any disposition of the pressures to which it can be subjected; and yet, compared with Figure 15 (which is the most generally used design), how simple and light! The weight of material may be roughly estimated at one-third (the beams braced, of course, not being included); and as to workmanship, the saving in that may even be greater, and susceptible of much more perfect execution; and the parts may be made removable for repairs with the greatest ease and safety, which is a matter of primary importance when wood is the material employed; at the same time, the liability to decay and failure is considerably lessened by the nature of the joint to be used, and by their diminished number.

CHAPTER II.

APPLICATION OF THE TRIANGULAR BRACING TO
VARIOUS STRUCTURES.

THE following view of the action of a bracing will often be found useful, in investigating the pressures brought into action :

In all braced structures, though two longitudinal pieces, at least, are necessary, yet one only of these may be considered as the primary member, all the other parts being looked upon as the means of bracing. Thus, for example, in Figure 23, the beam ab requires bracing; arrange, then, as shown in the figure, a series of pairs of braces along it; each of the triangles adn , nee , etc., is thoroughly braced, and may, therefore, be represented as solid plates. Now, the application of weights to the beam ab will give it the curved appearance represented in Figure 24, and this change is necessarily accompanied by the approachment of the apices, d, e, f, g , of the braced elements to one another, and the dotted lines, which were vertical in Figure 23, become here radii of curvature. If we prevent these changes by the insertion of strut-pieces between the apices, as seen in Figures 25, 26, etc., a braced structure will be produced. And when the braces are applied beneath, or when the upper beam is considered as the braced member, the intervals between the apices, on the addition of weights, would tend to increase, which must therefore be counteracted by the application of *ties*, as seen in Figure 27.

The action of bracing upon *flexible* arches, when viewed in this manner, becomes very evident; and, as this is one of its most important applications, it deserves particular consideration. In Figure 28, PaQ represents a flexible arch. When a load is placed on the crown, that, of course, is flattened and depressed, and the haunches are raised and further bent; and

when the weights are placed on the haunches, opposite motions and effects ensue. Now, the arch may be broken in two ways : either from being so unequally loaded that a part of it is ruptured by being too much bent, or, when the arch is so braced as to be retained in its proper form, it is crushed by the pressures transmitted along it ; the load required to destroy the arch in the latter manner is very much greater than is sufficient in the former ; hence the great utility of bracing the flexible arch.

In investigating the application of bracing to a great variety of structures, the work will be much simplified by arranging them into different classes, though one class may, in reality, be but the extreme of another. Adopting this plan, and commencing with the simplest forms—

THE FIRST CLASS that claims our attention is that exemplified by Figure 29, being two *parallel* beams with the bracing between them, the whole acting as a girder. This, with the triangular bracing, is an excellent design for bridges, and also for the longitudinal framings of roofs, steamboats, etc., and for scaffolding and other framings.

THE SECOND CLASS comprehends all structures that act on the girder principle, but which have not the longitudinal beams parallel. The designs shown by Figures 30 to 34 are included in this class. In the first division of these—viz., 30–32—though less material may be required than by Figure 29, that advantage will sometimes be more than counterbalanced by the increased difficulty of execution. The second division, comprehending Figures 33 and 34, consists of structures not requiring *bracing*, as that is defined at page 1—the external form, without addition, being a braced one ; but, when large, they require trussing, and though they do not, therefore, come strictly within the province of this treatise, yet, as the simplest form of trussing that can be introduced is exactly that of the sim-

plest bracing, with this difference, that the parts are required to act in only one capacity, we may be excused for introducing them. Figure 33 is a design now much used for iron roofs; the reverse of this, shown by Figure 34, is an excellent design for a *rigid* suspension bridge. The author has never seen the latter employed with more than three supported points. Of course, to secure rigidity, the sides of the triangle must be maintained straight lines.

THE THIRD CLASS.—The structures included in this class are those which derive their strength from a single braced arch. Figures 35 to 42 are examples. These may be divided into three sub-classes—*first*, those having the bracing arranged along the intrados, but which have not the arches tied, as 35 and 36; *second*, those which are braced internally, and have the arches tied, as 37 and 38; and, *third*, those which have the bracing along the extrados, as 39 to 42.

THE FOURTH CLASS consists of arrangements of two or more parallel, or nearly parallel, flexible arches, each contributing its share of support, and the whole, by the introduction of bracing between them, acting, in a great measure, as a single deep rigid arch—Figures 43 and 44.

TABLE I.

TABLE OF SYMBOLS USED IN THE FOLLOWING PAGES.

w = Weight at each loaded point of the span.

N = Total number of weights or points.

$W = N w$ = Total load.

s = Distance between the weights or points.

$S = N s$ = Total length or span.

P = Pressure acting in a given line, $a b$ —see Figure 45.

V = The vertical element or component of pressure P , $= b c$.

H = The horizontal element of pressure P , $= a c$.

$\theta = \angle a b c$ = The angle at which the given line, $a b$, is inclined to the vertical, $b c$.

$\varphi = \angle b a c$ = The angle at which the given line, $a b$, is inclined to the horizontal line, $a c$.

$$P = V \sec. \theta = H \sec. \varphi = \sqrt{H^2 + V^2}$$

$$V = P \div \sec. \theta = P. \sin. \varphi = P \cos. \theta = H \tan. \varphi.$$

$$H = P \div \sec. \varphi = P \sin. \theta = \tan. \theta.$$

CHAPTER III.

ON THE PRESSURES THAT MAY BE BROUGHT INTO ACTION IN CLASS FIRST.

THE best process for arriving at the amount of the pressures that may be brought into action in the several parts will be, first, to get the effects produced by each portion of the load separately, and then to find the maximum strains that can be produced by any arrangement of the weights; for the strains are not all the greatest when the whole length of the structure is loaded, as, by removing the weights from some of the points, the strains in some of the braces may, it will be seen, be increased. As an example of the method pursued in calculating for the results given in the following table, let us investigate the forces arising from one weight = w , placed at the point 4 of Figure 46. As the support contributed by each pier (from a property of the lever) must be inversely as the distance of the weight from it, the pier Q will support $\frac{4}{13} w$, while the other pier, P, supports the remaining portion of the weight, $\frac{7}{13} w$. The vertical pressure, $\frac{4}{13} w$, has therefore to be transmitted, through the bracing, to the pier Q; the real pressure, P, induced in the brace b will therefore be = $\frac{4}{13} w. \sec. \angle j 5 B$, = $\frac{4}{13} w. \sec. \theta$, and by means of that brace, the vertical element, V =

$\frac{11}{18} w$, is impressed on the point *k*. This point must be borne up by the brace *u*, which is consequently stretched with the strain of $\frac{11}{18} w$. sec. θ ; this is next transmitted through the brace *c*, and so on, till it is finally impressed at *Q*. The other portion of the weight, in like manner, causes strains of $\frac{7}{18} w$. sec. θ , in all the braces from the point 4 to the pier *P*, of compression in those which are inclined in the same way as brace *s*, and of tension in those inclined in the opposite direction. Proceeding in this manner, for each weight, we obtain the quantities marked against each brace in Figure 47; those written on the left side of the brace denote the strains of compression, and those on the right the strains of extension, which take place in the brace when the bridge is uniformly loaded, each number being the co-efficient of $\frac{w}{18}$. sec. θ ; of course, when two opposite pressures would act in a brace, the resulting strain, in the brace, is their difference; thus in brace 5 *j*, the strain, for a uniform load, is $(+25-16) \cdot \frac{w}{18}$ sec. $\theta = \frac{9}{18}$ sec. θ , which is + or a compressive strain; in this way the quantities in column second of Table II. are found. But referring again to the brace 5 *j*, we see that the tensive forces are produced by the weights 1, 2, 3, and 4; consequently, if these be removed, the full compressive strain of $25 \frac{w}{18}$ sec. θ will act in the brace, which is the maximum strain of compression that can possibly be induced in it by any disposition of the weights; on the other hand, if all the weights, excepting those at 1, 2, 3, and 4, be removed, there will be no compression induced in the brace 5 *j*, and therefore the total tensive strain of $61 \frac{w}{18}$ sec. θ will come into play; and this is the maximum of tensive strain that can take place in it; in this way the third and fifth columns of the table are calculated.

The maximum strains are those we have to attend to in fixing the strengths of the braces and of their joints; thus, in brace *w*, it is a tensive strain we have chiefly to guard against, whereas the required tensive strength of the brace *b* is very moderate, but it must be capable of withstanding four times as great a compression.

TABLE II.

1.	2.	3.	4.	5.	6.
Brace.	Strains when all the weights are on.	Maximum Compressive Strain by uneven Loading.	Weights removed to produce the Strains in Column 3.	Maximum Tensile Strain by uneven Loading.	Weights removed to produce the Strain in Column 5.
<i>e</i>	+ 81	+ 81	All on.	Never any.	
<i>d</i>	+ 63	+ 64	1.	— 1	All but 1.
<i>c</i>	+ 45	+ 49	1, 2.	— 4	All but 1, 2.
<i>b</i>	+ 27	+ 36	1, 2, 3.	— 9	1', 2', 3, 4', 5, 4.
<i>a</i>	+ 9	+ 25	1, 2, 3, 4.	— 16	1', 2', 3', 4', 5.
<i>t</i>	— 9	+ 16	1', 2', 3', 4', 5.	— 25	1, 2, 3, 4.
<i>u</i>	— 27	+ 9	1', 2', 3', 4', 5, 4.	— 36	1, 2, 3.
<i>v</i>	— 45	+ 4	All but 2, 1.	— 49	1, 2.
<i>w</i>	— 63	+ 1	All but 1.	— 64	1.

The numbers in the Table are the co-efficients of $\frac{w}{2N}$ sec. θ .
 $= \frac{w}{18}$ sec. θ , here, as $N = 9$.

From the foregoing and similar data, the following formulæ are deduced, the load being applied above :

The maximum compression of the last brace (1 Q, Figs. 46 and 47) is $= \frac{(N-0)^2}{2N} w$. sec. $\theta = \frac{N}{2} w$. sec. $\theta = \frac{w}{2}$ sec. θ .

The maximum tension in *m* 1, and compression in 2 *m*, $= \frac{(N-1)^2}{2N} w$. sec. θ .

The maximum tension in *l* 2, and compression in 3 *l*, $= \frac{(N-2)^2}{2N} w$. sec. θ .

And putting *n* to represent the number of the *pair* of braces as counted from the pier, along the lower beam, the general formula will be $\frac{(N-n)^2}{2N} w$. sec. θ .

When *N* is even, the formula for the central pair of braces will be $\frac{(N-\frac{N}{2})^2}{2N} w$. sec. $\theta = \frac{N}{8} w$. sec. $\theta = \frac{w}{8}$ sec. θ .

Carrying the calculation beyond the centre, we obtain the maxima of the other strains that can act in the braces.

We see, then, that the extreme braces require to be four times as strong against compression as is necessary for the central ones, when N is even, and very nearly four times when N is odd and large. And starting with the value $\frac{w}{8}$ sec. θ , at the centre (supposing N even), the tensive strains in the braces parallel to a , Figure.46, gradually diminish as they approach Q , where they vanish, but increase as they approach P , where the last amounts to $(N-1)^2 \frac{w}{2N}$ sec. θ ; and in these braces the compressive strains are also $= \frac{w}{8}$ sec. θ , at the centre, but gradually increase as they approach Q , where they amount to $\frac{w}{2}$ sec. θ , and diminish as they approach P , where the compression in the last is only $\frac{w}{2N}$ sec. θ . The strains in the braces parallel to s , of course, diminish and increase in the opposite directions.

N.B.—These maxima are obtained on the supposition that the whole value of w may be removed from any point, but, as the weight of the bridge itself is included in W , and cannot be altered in its distribution, the effect of uneven loading, as given by the above formulæ, will be rather exaggerated, particularly towards the centre of the span.

The maximum strains of the longitudinal beams take place at the centres of their lengths, where they amount to $^* \frac{s}{2D} \cdot \frac{W}{4} = \frac{s}{8D} \cdot W$, D being the depth of the framing. This amount of strain is compressive in the upper and tensive in the lower beam. The strains diminish towards the ends, and at the last *bays* they amount to $(\frac{N}{2} + \frac{N-2}{2}) w \tan. \theta = (N-1) w \tan. \theta$ of compression in the upper beam, and to $\frac{Nw}{2} \tan. \theta$ of tension in the lower, θ being the inclination of the braces with the vertical.

And if n' denote the number of the bay, counted from either

* S = span or length.



end, we have the following formulæ for the strains in the longitudinals (the load being on the upper level, and the terminal braces being struts):

$n' (N - n') \tan. \theta. w$, for the compressions in the upper beam; and

$\{n' (N - n') - \frac{N}{2} + n'\} \tan. \theta. w$, for the tensile strains in the lower beam.

When the load is at the lower level (and the terminal braces are struts), the strains will be as above, except that, for the lower beam, each will be $\frac{1}{2} w \tan. \theta$ less.

NOTE.—Turning to Figure 47, we observe that, when the weights are placed on the upper beam, the maximum strains producible in any two braces which meet at the lower beam have equal vertical elements; this necessarily results from the lower beam being horizontal; consequently, in the form of bracing shown by Figure 21, the same equality of the *vertical elements* must also exist.

If the load were placed on the lower beam, the foregoing observation would apply to each pair of braces which join at the upper beam. The formulæ, etc., which have been given, are calculated on the supposition that the load is applied at the upper beam, and they would therefore require to be slightly modified to suit the case of a suspended load.

CHAPTER IV.

ON THE PRESSURES THAT MAY BE BROUGHT INTO ACTION IN CLASS SECOND.

As a representative of this class, take Figure 49. Here, the distance between the points of support being uniform throughout the span, the angle θ for the braces is variable.

The maximum compression of the extreme braces, $a c$ and $b l$, takes place when all the weights are on, and, as half W is upheld by each, the strain will be $= \frac{W}{2} \sec. \theta$. But the next braces will not be strained with so great a tension as would take place in a bridge of class first with the same depth, $c n$;

being relieved from a portion of their duty of imposing the whole of $(N-1) \frac{2w}{2N}$ on the points c and l , by the inclined action of the upper beam. This will be made more evident by the following explanation of Figures 50 and 51; in these the depths at c , and the values of s , are the same, and also, therefore, θ for the braces. Let ag represent in each figure the upward resistance of the pier, $= \frac{w}{2}$, then will ac represent the compression induced in the brace ac ; resolving ac into the directions, bc and cd , of the other brace and of the upper beam, the tension produced in the brace, bc , is $= cb$ in Figure 50, but only $= ce$ in Figure 51. Further, we observe that the compressions in the upper beams of the Figures 50 and 51 are ab and ae respectively; the tension of the lower beam, beneath c , is the same in each, and $= gc$.

The strains at the centres of the longitudinal beams in Figure 49 are the same as for a bridge of class first having the same depth as it at the mid-span. Let us compare the strains at the extremity of the latter, represented by Figure 52, with the same of 49, represented by Figure 51. Make ag the same in each, and let it represent $\frac{w}{2}$, then ac will measure the compression in the extreme brace; that in 52 is the least. The tension of the lower beams is measured by gc in each figure, and is greatest in 51. Resolving ac into the directions cd and bc , cf is the resulting compression in the upper beam in each figure, and it is evidently the least in 52. The resulting tension in the brace cb is represented by ce in each figure; the comparative amounts are shown in Figure 51 by ce and cv , and ce diminishes as the angle acd increases, vanishing as that becomes $= 180^\circ$.

The advantages, then, of a design of class second over a similar one of class first having the same depth at the middle may be stated as reducing the strains in all the braces, except the extreme one at each end (which is rather more strained, becoming, in some degree, a continuation of the upper beam)—the central ones, however, are little altered—also as reducing

the lengths of the braces towards the extremities; and as rendering the longitudinal beams more uniformly strained throughout their lengths, being equally so with class first at the centre, but more so than in it at the extremities. In fact, as will afterwards be seen, this class is a step towards some forms of class third; it becomes identical with them when $a c$, Figure 49, forms a continuation of the curve of the upper beam (see Figures 37 and 38), or when a pressure is applied at c in the manner of an abutment, in the direction $c d$, as indicated by the dotted arrow. (See Figure 35.)

Though the triangular forms, shown by Figures 33 and 34, as stated in chapter second, are not *braced*, and consequently do not *demand* our attention, yet, on account of their utility and the reasons there given, we will introduce them here.

The weight at point 2, Figure 53, must cause equal strains in $1 a$ and $3 a$, as the angles $1 a 2$ and $3 a 2$ are equal, and $1 a$ and $a c$ form a straight line; so that half the weight 2 is transferred to point 3, and, along with the weight at 3, is impressed on b , the total weight, $\frac{3w}{2}$, at b is distributed, so that $\frac{w}{2}$ is carried to 1, and $\frac{2w}{2}$ to point 4 (see note at end of chapter); the total weight accumulated at point 4 is $= 3 w$, one w being received from each of the points b and d , and this is impressed on c , and thence transmitted, equally, to 1 and $1'$.

Now, the rule, derived from a property of the lever, that a weight at 2 is upheld by the piers, 1 and $1'$, in the proportion of 5 to 1, holds good here; for though in the first effect of the weight at a only $\frac{w}{2}$ is carried to pier 1, yet at each succeeding point another fraction will be transmitted to 1, by the rod $c 1$, until we arrive at the lowest point, e , where the remnant of $\frac{w}{2}$, amounting to $\frac{w}{3}$, will be equally divided between 1 and $1'$, so that $\frac{w}{6}$ will be the only portion of the weight, w , at point 2, which the pier $1'$ can receive.

After what has been said, little explanation of the Figures

55 and 56 will be required. In these, for the sake of having simpler numbers, the length of the side A C is made a multiple of the central depth h C. The numbers (co-efficients of w) attached to the parts are calculated on the foregoing principles, and give the real strains or values of P, which take place from a uniform loading. In this form of structure, the maximum strains are produced when all the weights are on. Of course, when, to serve for a roof or other purpose, the design is inverted, as Figure 57, the strains, with very trifling and evident exceptions,* are all the same in degree, but of opposite character.

NOTE.—In Figure 54, a weight at b , $= w$, $= ab$, is distributed thus: It is resolved into $b c$ and $b d$; and further resolving $b d$ into a horizontal and a vertical pressure, $b x$ is the latter, which is, therefore, the portion of w , or $a b$, that the rod $b h$ receives; and resolving $b c$ similarly, we get $c e$, $= a x$, as the vertical, which represents the portion of w borne by the rod $b g$. The triangle $b x d$ is evidently similar to the triangle $a x f$, $\therefore b x : x d = a x : x f$, and alternately, $b x : x a = d x : x f$. But triangle $d a f$ is similar to $g b h$, therefore the lines $a x$ and $b a$, drawn from corresponding angles, and at right angles with the opposite sides, must divide these sides similarly, therefore $d x : x f = g a : a h$, and consequently $b x : x a = g a : a h$. The fact that the weight at b is distributed to h and g in the proportion of $g a : a h$ might have been anticipated from the property of the lever.

$g a : a h = \tan g b a : \tan h b a$. $b d = \frac{ag}{gh} w \sec. a b h$. And $b c = \frac{ah}{gh} w \sec. a b g$; the resulting horizontal pressures are, of course, the same for each direction, and are $d x$ and $b e = b d. \sin a b h$.

*Arising from the weights being on the slant sides, instead of being attached to A B, which would be the true arrangement in Figure 56 inverted, and consequently relieving each perpendicular piece of $1 w$.

CHAPTER V.

ON THE PRESSURES THAT MAY BE BROUGHT INTO ACTION IN CLASS THIRD.

WE now come to consider the case of a braced flexible arch. The form given to the arch should, theoretically, be the curve of stability for the fully loaded arch (in the investigations it is assumed to be this, so that, in the internally-braced arch, no action will be induced in the bracing when all the weights are on). This curve, in general, will be found to lie between a catenary and a parabola; but, as the quantity of the curve generally used is not great, a circular arc may be adopted, as it very nearly coincides for almost 120° with the theoretical figure. When an arch of a greater number of degrees is required, and the structure is of a light character, it is generally advisable to use the parabolic form. But in a *braced* arch we may depart considerably from the correct curve without much evil; some of the braces might require to be made stronger, and a trifling variation would take place in the strains of some of the other parts.

When the load is uniformly distributed on the arch there is no bracing required, and consequently (if the line of pressures remains in the arch*) there is no action induced in the bracing. When the load is *suspended* from the arch by means of the braces, of course there are tensive strains produced in them.

As, in the investigations, the values of w and s are constant, and the weights of the arch and other parts are not considered, the curve of equilibrium will be a parabola, or rather will be polygonal, having its angles arranged in a parabola.

* The curious properties which occasion this restriction will be found mentioned at page 25.

THE ARCH WITH INTERNAL BRACING.

The following fundamental principle of investigation is capable of very general application ; it might have been introduced in Chapter III., but the method there adopted is a simple one deducible from it for that particular case, viz., where the longitudinal members are all horizontal. From a law of the lever, we know that the vertical elements of the pressures upon the abutments, resulting from a weight, w , placed at the point c of Figure 59, must be inversely as the horizontal distances of c from them ; here, therefore, the pier P will bear $.75 w$, while the pier Q supports the remaining portion, $.25 w$. The vertical pressure of the weight w at c must be *resolved into the directions $c P$ and $c Q$* , the portions of the bridge, $P b c s$ and $Q f c t$, acting as instruments for the transmission of the respective pressures to the piers P and Q ; in acting as such, let us trace the strains produced in the parts. The pressure in $c P$ will be resolved into the directions $c b$ and $c s$, and the vertical elements, or the values of V of these two pressures, since they both act downwards, must, added together, equal $.75 w$; the pressure in $c s$ is at s resolved into pressures in $s P$ and $b s$; that in $b s$ is a tensive strain, and must, since $s P$ is horizontal, contain the same vertical element as the pressure in $c s$. *The resultant of the two strains, in $c b$ and $b s$, conveyed to b , must have the direction $b P$, and possess a vertical element $= .75 w$;* for the resultant of the whole, acting in the line $c P$, passes through P , and as the pressure $s P$ is horizontal and passes through P , the remaining component pressure, $b P$, must also pass through P , and must contain the whole vertical element. The pressure $b P$, is resolved into $b a$ and $b r$, and $b r$ is resolved into $r P$ and $a r$; the resultant of the compression in $b a$, and the tension in $a r$, must have the direction $a P$, and contain a value of $V = .75 w$. The final resultant of the pressure in $a P$, and the horizontal pressures, arising at the points s and r , must have the direction $c P$, and its vertical element $= .75 w$. Returning to the point c , the pressure in

$c Q$ is resolved into the directions $c d$ and $c t$, and since $c d$ acts upwards, the vertical element of $c t$ must *exceed* V of $c d$ by $.25 w$, as that is the downward vertical element of the resultant to the pier Q ; the pressure in $c t$ is resolved into $t Q$ and $d t$. The resultant of the two pressures conveyed to d , by the compression in $c d$ and the tension in $d t$, must have the direction $d Q$, and contain a vertical element $= .25 w$. The resultant in $d Q$ is resolved into $d e$ and $d u$; but we need not proceed further in tracing the transmission of the pressures to Q ; sufficient has been said to explain the principle. We see that (when the lower longitudinal member is horizontal) the resultants at the various points of the arch, at either side of the weight, are *directed to the pier or spring of the arch* at that side; and all those at one side of the weight have the *same vertical element* or value of V , which is eventually imposed on the pier at that side.

We will now give, in considerable detail, the investigation of a simple example having five points or weights, shown by Figure 58. Let a weight be placed at the point a , and represented by the line $w a = 1$; this will be resolved in the directions of the piers P and Q , and the amount of the pressures produced in these directions will be $= a f$ and $a m = a o$; the vertical elements of these will, as given by the lever rule, be $= 0.9$ and 0.1 or $= f n$ and $n m$; the horizontal element or value of H is the same for each, and $= n a$. The resultant $a o$ is resolved into $a l$ and $a t$; $a t = a' g$ into $g u$ and $r g$; and so on throughout.

Table III. gives the results for the various parts, from the action of one eccentric weight at a .

TABLE III.

	Vertical Elements or Values of V.	Real Strains or Values of P = V sec. θ .
wa = vertical weight at a	1.000	1.000
af = resultant to pier P.....	.900	1.260
$ma = ao$ = resultant to pier Q.....	.100	.900
na = value of H of each resultant..	—	.895
Pressure in ab (measured by $a t$)....	.300	.582
“ in ag (measured by $a t$)....	.400	.563
“ in gu	—	.580
“ in gb , tensive.....	— .400	— .4364
Resultant $b Q$100	.333
bc050	.255
$b h$150	.165
$h v$	—	.125
$h c$	— .150	— .161
Resultant $c Q$100	.216
cd034	.173
ci066	.071
iw	—	.055
id	— .066	— .0726
Resultant $d Q$100	.168
de075	.1455
$d j$025	.0273
$j x$	—	.036
je	— .025	— .035
Resultant $e Q$100	.140
$Q k$	—	.099
$Q y$100	.100
Horizontal thrust = $na = yz = gu + hv + iw + jx + Qk$.		
“ “ = 895 = .580 + .125 + .055 + .036 + .099.		

Tables, similar to this, being constructed, one for each weight, it is easy to discover by what arrangement of the weights a maximum strain in any part is produced, and the amount of that maximum. Thus, taking the brace $b h$, we have produced in it:

TABLE IV.*

	Real Strains or Values of P.	Values of V.
By weight a ..	$\pm .165$ }	$\pm .150$ }
“ “ b ..	$\pm .495$ } = + .660	$\pm .450$ } = + .600.
“ “ c ..	$-.3674$ }	$-.334$ }
“ “ d ..	$-.2200$ } = — .660	$-.200$ } = — .600
“ “ e ..	$-.0726$ }	$-.066$ }

* These values are taken from tables similar to Table III., but it would be useless to give them here. The sign $+$ indicates a compressive, and $-$ a tensive strain.

From Table IV. it is evident that when the weights a and b alone are on, there is a maximum *compression* produced in the brace $b h$; and when these are removed, and the other weights, c , d , and e , are placed on the arch, a maximum strain of *tension* takes place of exactly the same amount. When all the weights are on (agreeably with a rule previously given, since the contour of the arch is such as to render it equilibrated under the whole load), the compressive and tensive strains neutralize each other, and no effect is produced in the brace.

The maximum compression for any brace is equal to the maximum tension for the same; hence, calculating the compression is sufficient, as the result is the measure of the necessary strength to be given to the brace to resist both compression and extension.

At any point of the arch (c , Figure 59) we can easily find, by the lever rule, the value of V for each resultant from that point to one of the piers (Q), caused by each of the weights (c , b , a) situated at and to the other side of the point (it is .25, .15, and .05); now, the greatest compound resultant that can act in the same line, will evidently have the value of V in it equal to the sum of the values of V above mentioned (thus, $.25 + .15 + .05 = .45$), and its amount will be V multiplied by the secant of its angle with the vertical ($P = .45. \sec. \theta$). Having thus the compound resultant, in value and direction, all we have to do, in order to find the maximum strain ($= 1.097$) in the first brace (ct) from the point, is to resolve it into the directions of the chord of the arch (cd) and of the brace. But the following formulæ offer very ready means of calculating the necessary strengths of the braces, as they give at once the value of V for the maximum strain in each.

The formula for the end pairs of braces (as $a r$, $r b$, and $j z$, $z i$, of Figure 59) is $\frac{1w}{2N} \cdot (N-1) = \frac{1(N-1)}{2N} \cdot w = V$.

The general formula, putting n to denote the number of the *pair* as counted from the pier, is

$$\frac{n.w}{2N} \cdot (N-n) = \frac{n(N-n)}{2N} \cdot w = V.$$

The second column of the following table gives the formula for each of the first six pairs of any bridge. The last two columns contain the results for the case of Figure 59, wherein $N = 10$.

The values of V for the braces are not altered by a change of the rise or span of the arch, while N and w remain the same; hence having calculated the vertical elements for the given value of N (as is done in Figures 60 and 61), it only remains to multiply each by the particular value of w . sec. θ , in order to obtain the strains, or values of P .

TABLE V.

Value of n or No. of the Pair of Braces.	Equations for the Value of V , a maximum in the Braces.	Values of V when $N = 10$, as in Fig. 59.	Values of $P = V$. sec. θ or Real Strains for Fig. 59, wherein the Rise to Span = 1 : 4.75.
1.	$\frac{1 (N-1)}{2 N} w$	$0.45 w$	$\begin{cases} a r = 0.702 w \\ r b = 0.496 w \end{cases}$
2.	$\frac{2 (N-2)}{2 N} w$	$0.80 w$	$\begin{cases} b s = 0.882 w \\ s c = 0.836 w \end{cases}$
3.	$\frac{3 (N-3)}{2 N} w$	$1.05 w$	$\begin{cases} c t = 1.097 w \\ t d = 1.085 w \end{cases}$
4.	$\frac{4 (N-4)}{2 N} w$	$1.20 w$	$\begin{cases} d u = 1.240 w \\ u e = 1.235 w \end{cases}$
5.	$\frac{5 (N-5)}{2 N} w$	$1.25 w$	$\begin{cases} e v = 1.286 w \\ v f = 1.286 w \end{cases}$
6.	$\frac{6 (N-6)}{2 N} w$	$1.20 w$	$\begin{cases} f w = 1.235 w \\ w g = 1.240 w \end{cases}$
n .	$\frac{n (N-n)}{2 N} w$		

Each weight produces compression throughout the arch; therefore the compression will be a maximum when all the weights are on. The vertical element of the compression at the springing or foot will evidently be the half-load; and for any point, it will be half the load above the level of that point.



Of course, the real compression at any point in the arch is = the vertical element multiplied by the secant of θ there.

It is scarcely necessary to mention the well-known fact that the horizontal element is, for an equilibrating load, uniform throughout;* and that the horizontal thrust for the wholly loaded arch is the measure of the maximum compression of the arch at the crown.

The strain in the lower beam, when it acts as a tie to the arch, is greatest when all the weights are on; the tension is then uniform throughout the length, and equal to the horizontal thrust of the arch, or to the compression in the arch at the crown.

When the lower beam does not act as a tie† to the arch, but simply as an adjunct to the bracing, it undergoes, principally, compressive strains, and these are rendered a maximum for one half-length of the beam, when the weights above that half are removed. Under such a loading, the compressive pressures in the lower beam increase as they approach the pier at the unloaded side, where the compression *may* amount to about one-half the value of the horizontal thrust for the wholly loaded arch. (See NOTE.)

SCHOLIUM.—Reviewing what has been said, we observe that in class first the strains in the braces increase towards the extremities, and that there, for the same brace, the tensive and compressive maxima are very different in value; whereas in the

* When the load is not an equilibrating one, the braces come into action, and transmit parts of the horizontal pressures to the piers; but if a vertical section be made at any part of the span through the arch, bracing, etc., then the sum of the horizontal elements, in the various parts of the section, is a constant quantity for any one loading. From this property, the line, which in the subsequent pages is called the *line of pressures*, has been named, by Mr. W. H. Barlow, the *line of equal horizontal thrust*; it is the resultant of all the pressures acting in the structure, with the exception, when the arch is a *tied* one, of the strains occasioned, by the horizontal thrust of the arch, in the tie-beam.

† A tied arch of this form is evidently the extreme of the change of class first (Figure 46) into Figure 49 of class second. It might, indeed, without impropriety, have been included as a variety of the latter class.

form treated of above (Figures 59, etc.), the central braces are those most strained, and every brace has its compressive and tensive maxima equal. Now, there is an intermediate form between these two, and its parts will undergo strains of a medium character; such is Figure 49 of the second class, and it is possible so to modify that form as to secure a very equable distribution of the strains, both in the braces and in the longitudinal members.

NOTE.—In the form Figure 59, with the lower beam not acting as a tie—when all the loading is on, the line of pressures, of course, corresponds with the line of the arch. But when the load is only placed on a space at the crown, the line of pressures resulting is shown by the dotted line in Figure 62. Where the line rises above the arch, the lower beam, immediately beneath that part, acts negatively or as a tie; and when the line falls within the arch, the lower beam, at that part of the span, acts positively, or as a strut, conveying part of the compression to the abutment. These observations upon the relation between the position of the line of pressures, with regard to the arch and the nature of the pressures induced in the lower beam, apply universally. Figures 63 and 64 give an idea of the positions the line may assume when half the length of the arch is loaded.

THE ARCH WITH EXTERNAL BRACING.

In this variety (exemplified by Figure 65), the line of pressures for the fully-loaded arch does *not necessarily coincide* with the arch; for it has been demonstrated by Mr. W. H. Barlow (in an excellent and valuable paper read at the I.C.E., July, 1847, a report of which is given in the *Civil Engineer and Architect's Journal*, Vol. X., p. 211) that the real curve of action is that which produces the least horizontal thrust—namely, that curve (parabola here) which rises to the neighborhood of the extrados, which is here the upper beam, as shown by the dotted line.*

* The curve or line of pressures *may* extend beyond the structure when

Nevertheless, it will be proper to give sufficient stability to the abutments, and strength to the arch and bracing, to withstand the pressures produced when the central line of action coincides with the arch, as such a coincidence would take place if the upper part, *c*, should become injured or materially shortened. If the part *c* become slightly shortened (from the great compression), the crown of the line of pressures will descend proportionately.

Unlike the foregoing case of Figure 37, of internal bracing, the values of *V* for the braces, when *N* is given, are not constant.

CHAPTER VI.

ON THE PRESSURES THAT MAY BE BROUGHT INTO ACTION IN CLASS FOURTH.

THE introduction of bracing between the parallel, or nearly parallel, flexible arches of this class renders the whole structure in its action a single rigid arch. The dotted line in Figure 66 shows, as stated in last chapter, the line of pressures (*W* being all on) as long as the structure remains perfectly without change; but the centre of the upper arch and the ends of the lower one, from their compressibility, will become somewhat shortened, and from these changes the line of pressures will sink at the crown, and not descend so far at the springings, becoming, in fact, more nearly identical with the midway line; it approaches this as the pressures become more nearly equal to the ultimate strength of the compound arch; and if one arch be stronger than the other, the line will lie nearer to it, as it recedes from an arch in proportion to the compressibleness thereof.

that is braced; but if the parabola rise above the upper beam, the action will partake, in a proportionate degree, of the nature of that of the girder—the *arch* beneath the part being then in a state of *tension*.

Such are the deductions in theory; but to ensure these, we must have a structure far more perfectly framed than is practicable. Let us see what the shortcomings of practice may entail.

If, at the time of the erection of the structure, or from after-causes, such as decay or other damage, the abutments, at the different footings, be not all applied with the same accuracy, disturbing causes will be introduced; and when the faultiness of an abutment is so great that the arch there is relieved from pressure, it becomes only a part of the bracing of the other arch, through the spring of which the line of pressures must necessarily then pass. From this it is evidently proper that one arch should be made sufficiently strong to carry the whole load, *unaided* by the other as such.

CHAPTER VII.

WE will now investigate the effect of inclination, or the value of θ , of the braces, on economy. Let Figures 67 and 68 represent two structures, having the same span and depth of framing, and loaded to the same amount. Referring back to Chapter III., we find that the strains in, and therefore the necessary strengths of, the braces are proportionate to $W. \sec. \theta$; and as W is here constant, the requisite strengths are proportionate to $\sec. \theta$. The length of a brace in each figure, since the depths are alike, may be taken to represent $\sec. \theta$. Let the values of $\sec. \theta$ be equal to b and b' .

The requisite sections* of the braces may be assumed to be proportional with the strains, and therefore also with $\sec. \theta$, or

* Of course this is not strictly correct with regard to compressive strains (and therefore θ will be a very little less than deduced). But while the number of systems remains the same, the disparity will not be important. And in the case of an increase in the number of systems, the more significant error that might take place will be moderated in practice by the braces being riveted, bolted, or trenailed together at the crossings.

as $b : b'$. The total lengths of the bracings are evidently as $16 b : 8 b'$.

Now, the total weights of the bracings are to one another in the same proportion as the total lengths multiplied by the respective sections, or as $16 b \times b : 8 b' \times b' = 16 b^2 : 8 b'^2 =$ weight of bracing of Figure 67 : weight of bracing of Figure 68. Or, as a general rule, the weight of a bracing is proportional with $2 N \cdot (\sec. \theta)^2$, but $N = \frac{S}{2 \tan. \theta}$ which is proportionate to $\frac{1}{2 \tan. \theta}$. Substituting this for N , we have *the weight of the bracings proportionate to $\frac{(\sec. \theta)^2}{\tan. \theta}$, which is a minimum when $\theta = 45^\circ$.*

Though the minimum of weight of bracing is arrived at when θ is made $= 45^\circ$, yet we may vary this angle considerably without much increase of the weight, as will be seen from Table VI.

TABLE VI.

Values of θ .	Values of $\frac{(\sec. \theta)^2}{\tan. \theta}$.
10°	292.40
15	200.00
20	155.60
25	130.55
30	115.50
35	106.40
40	101.55
45	100.00
50	101.55
55	106.40
60	115.50
65	130.55
70	155.60
75	200.00
80	292.40

But when, with $\theta = 45^\circ$, the distances between the supported points are *very much* too great, the difficulty is not to be overcome by lessening the value of θ , but by means of the intermediate props shown in Figure 69, or by using two or more *systems* or ranges of bracings, as shown in Figures 70, 86, and 87. We need only make this remark, at present, upon the theory of systems: that as the value of w is reduced as much as the number of systems is increased, the total weight of the bracing (supposing the section to represent the strength of a brace) remains the same as for a single system; but the quantity of workmanship will be much greater.

PLATE BRACING.

But it would be difficult to define the limits of this description of bracing. In one direction it includes the simple beam;

in another, the cylindrical tube, and even hollow spherical forms cannot be excepted as varieties of it. Among the more obvious cases of its employment, we may mention the feather on a casting, the sides of a box, the hull of a ship, and the floor, walls, and roof of the majestic tube of Stephenson and his coadjutors. It is the bracing employed in nature, and that in all its varieties. In the plate and shell form, we see it in the beak of a bird, in the bones of the cranium, and in animals with an external skeleton, as the crab, etc.; as a *feather* strengthening a plate, we view it, most conspicuously, in the keel of the breast-bone of the falcon tribe, and in the spine of the scapula of the land mammalia; and in the form of the tube, we witness it universally adapted to the wants of the animal and vegetable kingdoms—the bones of the extremities, the quills of the wing, the humbler grasses, and the lofty bamboo, may be cited as familiar examples. We shall, however, confine ourselves to the consideration of it in its simplest form, and endeavor to compare it with the brace or *linear* method.

We have observed above that, to derive the *plate* from the *linear* bracing, the lateral width must shrink into the thickness of the resulting plate; consequently, it has comparatively little lateral stiffness, and would readily warp or *pucker* wherever the compressive pressures were considerable; fortunately, however, this can be remedied with a moderate addition of material in the form of featherings, and of easy application in the case of the material (malleable iron) which would generally be employed for this kind of bracing.

At those parts where the *tensive* strains considerably predominate, we may expect to find the plate *stronger* than the linear form, from the following considerations: Let Figure 71 represent a bracing of numerous series, and let a tensive strain act in *a b*. Now the strain is only borne by the parts lying in the direction of *a b*; but when the systems become infinite and the whole solid, there are, as it were, two plates—one resulting from the braces lying in one direction, and the other from those in the opposite one; and each of these plates serves to bear the

strain in $a b$. And, further, the direction of a strain is not confined to a particular angle.

The comparison, then, may be enunciated thus: "The plate bracing possesses much greater strength against tensive strains, but less against compressions, than the linear." And, as a corollary, we must infer that, generally, the plate form is better suited for shallow than for deep bracings.

The general distribution of material in the plate bracing must be subject to the laws regulating it in the other methods; thus, in the first class, the plate must be made more substantial towards the extremities; and in Figure 37 of the third class, the central part must receive the greater share of material. Figures 72 and 73 give an idea of the strains in class first; the lines show the general action of the pressures, and the degree of shade produced is an index to the strength required. But these diagrams are, of course, made much simpler than would really occur, and they are only adapted to illustrate the action of a uniform loading. In Figure 72, at the pier P, the perpendicular lines indicate that the pressures impressed at a by the lines of suspension, shown in Figure 73, are supported from beneath; but at pier Q these are supposed to be received by a higher part of the pier from the edge b , as is done by means of the ball-rollers in the Conway and Britannia tubes.

Part the Second.

CONSTRUCTION.

CHAPTER I.

THE VARIOUS BRACINGS REQUIRED IN DIFFERENT STRUCTURES.

BESIDES the longitudinal vertical bracings, upon which depends its strength to support the loading, a bridge requires other bracings, and to the consideration of these we now proceed.

Let Figure 74 represent the skeleton of a bridge, composed of two vertical frames. That these may act with effect, they must be retained in straight lines, but there is the pressure of the wind, etc., tending to bend them laterally; this, therefore, must be sedulously guarded against; let us add, then, a series of horizontal braces arranged between the two upper beams, as in Figure 75. The upper beams are now rendered permanent in their position, and a roadway may be carried along them; the lower beams will be less steady, they may be moved by a lateral blast, but, as they undergo a tensive strain, they are in a state of stable equilibrium, and consequently return to their places. This wriggling motion would, however, very soon injure the structure, by loosening the joints of the vertical bracings to the upper beams; such a design is, therefore, inadmissible for a permanent structure; for such, the lower beams must be retained perpendicularly beneath the upper ones, which may be effected either by the introduction of bracing between them, similar to that used for the upper ones, or by the application of oblique transverse braces, as shown in Figures 76 and 77. The first of these methods will

be sufficient, without addition, for bridges having moderate spans compared with their widths; while the second may be trusted to in any case, but, of course, in it the horizontal bracing employed in the upper part will require to be of double strength, as it must resist the whole pressure of the wind. The common practice is to employ both of these methods in the same structure, and in long spans, if it do not interfere with the roadway. The practice is to be commended. Figure 77.

The principal recommendation of the first method, when used alone, is that it permits of the roadway being placed at the level of the lower beams, which is of the utmost importance in situations where the bridge spans over roads or navigable waters, and sufficient height could not otherwise be given for the passage of carriages or shipping beneath. Further, it requires little expense for parapets, and offers great facilities for the addition of a roof, or even of a second roadway, along the upper beams. But particular attention must be paid to carrying up the abutments, so that they may offer sufficient resistance to the outsides of the upper beams.

In the design, Figure 38, when the rise of the arch is very small and the span moderate, and when the arches have a considerable width of section (which may be conveniently given to them, if built up with deals, or constructed as tubes with wrought iron, as Figures 80 and 144), then a single series of horizontal braces along the lower part may be sufficient, and the roadway may be on that level. (This also applies to class first when the depth of the framing is small.) But when the structure does not comply with the above conditions, it will be necessary to have a series of braces between the arches also; for, as they are in a highly compressed state, a very moderate deviation from a horizontally straight line might bring about their rupture. But horizontal bracing being introduced between the arches, the road cannot be carried along the chord level; it must be raised to the level of the crown of the arch,*

* Unless some new arrangement be made in the design, as in the follow-

and the design then becomes that of Figure 37, and the roadway will also require bracing.

Most of the other structures of class third will be secure with two lines of horizontal bracings, and of these Figures 35 and 42 admit the roadway at the lower level. But when Figures 36 and 41 are of considerable span, it will be prudent to bestow three series on them.

In structures belonging to class fourth, each arch will require to be braced horizontally, and, generally, the roadway also.

PLATE-BRACING.

The forms of bridges to which the plate-bracing is more especially adapted are the tubular and the segmental. Figures 29, 35, 38, etc.

In the first, there is this important source of economy, that each side of the tube is useful in two capacities—first, as the bracing against pressures acting in its plane; and, second, as a longitudinal member against pressures acting at right angles to its face. In the form, Figure 35, the strains in the plate are more uniform throughout.

In the tubular form, let us consider, for an instant, the action under a load, confining ourselves, for the present, to

ing structures. Figures 78 and 79, which are examples of the use of plate-bracing, may be used when the rise is not great.

Figure 81 is an arrangement suitable for *linear* bracings, and there is less waste in it than would at first sight appear. The inside arches are kept perpendicularly over their chords by the transverse bracing, which occupies the central compartment throughout the whole length of the bridge; and cross connecting pieces or braces, throughout as great a length at the crowns as can be permitted, retain the outer arches parallel with the inner ones. In this design the rise must necessarily be considerable.

Figures 82, 83, and 84 show varieties of another arrangement; it is a compound of the first and third classes. The arch has the roadway suspended from it by iron rods, and, in 84, it is stiffened by separate braced frames, placed one on either side of it; bolts are passed through the whole wherever the arch crosses a part of the frames; the rise is made sufficient for the roadway to pass beneath the horizontal bracing between the upper beams.



the consideration of the bottom plate; this is stretched, but it will evidently be more so immediately below the vertical bracings, and rupture thereof would commence by the tearing of the edges. The same quantity of material collected into tie-beams, and attached to the bottoms of the vertical bracings, would offer a greater resistance; but then, on the other hand, an additional horizontal bracing would become necessary. Similar reasoning applies, and with greater force, to the top plate; and on the determination of which of these arrangements is the most economical depends, in a great measure, the question of the respective merits, for this case, of the plate and linear bracings when constructed of wrought iron; for other materials the plate form of bracing is not suited.*

The want of uniformity between the strains at the edges and those at the middle line of the plate, when it acts as a longitudinal member, may be obviated by giving it a slight convexity outwards, as in Figure 85. This may be done to the top and bottom plates, as, though it weakens them as bracings, sufficient strength, as such, will remain to counteract any lateral disturbing influences. But this must be looked upon rather as a refinement in theory than a practical improvement.

In the form of Figure 38, the plate-bracing may be modified so as to give lateral stability to the arch; this is shown more particularly in the design Figure 78, wherein the plate is double. The deep featherings in Figure 79 are designed to effect the same object.

The featherings alluded to at page — may be arranged on two plans: first, in perpendicular pieces, as in the Britannia tubes; and, second, radiating from each pier, as do the lines of

* Cast-iron may, indeed, be used for plate-bracing, and, since that form of bracing demands comparatively little tensile strength in the material, probably with good effect, at least where heaviness would not be a defect. Considerable skill must, however, be exercised when the bracing is composed of many parts.

compression in Figure 72; in this last arrangement they require to be of larger section to be equally effective, but, besides stiffening the plate, they carry a portion of the compressions to the piers. Perhaps, from various considerations, a combination of the two plans would be the best, the featherings being applied in the one manner to the inside, and in the other way to the outside of the plate.

CHAPTER II.

ON THE CONSTRUCTION AND APPLICATION OF THE BRACES.

WHEN the bracing is poly-systemed, the braces may be arranged on different plans; thus: All the braces of the same system, as shown by Figure 86, may be in the same plane, and the systems so near that the braces touch, notch, or halve into one another at the crossings. Or, as seen in Figure 87, all the braces lying in the same direction may be in the same plane, and those inclined the contrary way in an adjacent plane.*

In designing the joints of the braces, the following observation on the action of the bracing is of such great importance that, though indicated in the previous part, we again place it before the reader. That part of a pressure, in a brace, which is resolvable at right angles to the longitudinal member is, when that member is straight, *wholly* transferred to the other brace; and when the longitudinal beam is curved, the part transferred is *slightly* diminished or increased, according as the curvature is concave or convex towards the braces, when the longitudinal is compressed; and when it is stretched, the reverse is the case. And the only pressure resulting in the longitudinal is in the direction of its length.

* Or, as a variety of this second arrangement, suitable to a very broad longitudinal member, there may be *several* layers of parallel braces, inclined alternately in opposite directions.

WOODEN BRACES.

1. *Those requiring little or no iron.*—Braces of this character would generally be used in those situations only where iron-work might be difficult to procure. The simplest kind of jointing for them is that by means of trenails, as in Figure 88, or by dovetail notching, as in Figures 89 and 90. These methods are very suitable when the braces are thin or plank-shaped, as in Town's Lattice Bridge. But when a brace is liable to undergo great compression, its section should be nearly square or circular. For braces of a stouter character, the Figures 92 to 95 illustrate methods of jointing adapted to a longitudinal member constructed in a similar way to that shown by Figure 95. Modifications of these joints might be easily constructed, capable of correcting the length of the brace; but such a property is unnecessary, as there is no difficulty in the way of preparing the braces with the requisite accuracy.

2. *Those jointed with iron.*—The most appropriate way of applying wrought iron is in the form of straps, of which several varieties are shown by Figures 96 to 101. But generally the method illustrated by the succeeding Figures 102 to 117, of jointing the braces by means of CAST-IRON SOCKETS, is recommended by the author. These sockets are of two kinds; the first, including Figures 102 to 109, requires the end of the brace to be of a dovetail shape; the second, shown by Figures 110 to 117, to be square-headed. The first is best suited to braces formed of a wood that is easily split, the second for those of a tougher material.

The disadvantages attending the braces with the dovetail joints are, that they require stronger sockets, in order to resist their wedge-action; and that they might be sensibly lengthened by the compressibility of the dovetails allowing the ends to draw a little, unless they be wedged very tightly in, which would increase the former evil.

The only remark, since the subject is so fully illustrated by

figures, that it is necessary to make upon the construction of the square-headed braces is this : If the joint were formed as in Figure 116, there would be great danger of the piece a being wrenched off by any slight wriggling motion that might take place in the brace before the structure was perfected ; it is therefore constructed, as shown by Figure 117, so that there is no angle at b , which, by acting as the tooth of a crowbar, could split off the piece a ; but the slant of $b c$ must be slight, otherwise there would be the wedge-action objected to above.

Figures 109, 114, and 115 are forms of double sockets for the case, mentioned in the beginning of the chapter, of the braces of the same system being arranged in two planes ; see Figure 87. And Figures 102 to 106, and 110 to 112, are forms of single sockets, which may sometimes be found useful in transverse and other bracing. The other forms are for the case of the braces of one system being all in the same plane, as seen in 86.

The braces are retained in the sockets by short screw-bolts, either with or without covers.

The head of the brace and the inside of the socket should be coated with some preservative substance, such as a heated mixture of tar and pitch.

One great advantage attending the use of the socket-joint, and also participated in by most of the previous forms, is that if a brace becomes injured by decay or accident, it can readily be removed and replaced by a sound one.

The designs given are calculated for braces of moderate dimensions.

IRON BRACES.

To braces of iron the forms that may be given are very numerous ; but if we exclude those which do not admit of being individually removed, the number will be very considerably reduced ; and the best forms of sockets, such as Figures 118 and 119, are liable to this objection.

1. *Cast-iron braces* may have various forms of section, either feathered or tubular; but those of the latter kind are not removable. The design 124 is an excellent one; it possesses every desirable property, especially that of the pair of braces interchanging their pressures without the intervention of the longitudinal, the advantages of which were pointed at in the second paragraph of this chapter. But it cannot, without modification, be applied to a longitudinal member of the tubular class, as the bolts must not be inserted from the inside of the tube, for, should one break, it could not be easily replaced.

When tubular forms of the longitudinals are used, whether of cast or wrought iron, they must be furnished with sufficient featherings, single or double, to which the bracings are to be attached; the featherings may be placed either longitudinally or transversely.

Figures 125 and 126 show modifications of Figure 124 when it is connected with a tube. But when braces are applied to the sides of a longitudinal feather, the designs shown by Figures 132 and 134, which are suitable for the arrangement shown in Figure 87 of a poly-systemed bracing, possess considerable advantages.

2. *Wrought iron braces* may be of round or rectangular section, or rolled with a section possessing greater strength against compression, or they may be constructed of thin plates, strengthened by angle iron, or by being formed into pipes. Braces with solid ends may be jointed, as shown in Figures 131 to 134, for cast ones; but those with thin and broad ends will be most effectively joined to the main beams by being riveted to a projecting feather thereon (Figures 135 and 136).

On the subject of the respective merits of cast and wrought iron as the material of the bracing, the only general remark we shall make is that for very long braces the former, and for very short ones the latter, kind of iron is to be preferred.*

* Wrought iron may, however, be very properly employed for great lengths, if formed into tubes, or otherwise braced, especially when lightness is a consideration.

CHAPTER III.

ON THE CONSTRUCTION OF OTHER PARTS OF BRIDGES.

As the contents of this chapter are in a great measure supererogatory, the meagreness of the account of such important subjects needs no apology.

WOODEN STRUCTURES.

The longitudinal members.—When these are too long or too much curved to be formed of one piece, the best construction is to build them up with planks, using comparatively thin ones when the part is to act as a tie or is an arch of moderate radius, and thicker planks or deals when the part is straight and only subject to compression. In the latter case, care must be taken to bring the butting ends into forcible contact; indeed, it might even be advisable to place in the joints thin wedges of hard wood or iron, which could, when needful, be tightened up to the proper degree. But in arches formed of *thin* planks, if care be taken in the construction, the wedges may be dispensed with; the purpose will be effected, in a great measure, by the great friction, and this may be further increased by coating the planks with a hot mixture of pitch and tar, or by placing in all the joints a layer of strong brown paper dipped in boiling tar (as used by Messrs. J. & B. Green in their laminated timber arches); but the most important benefit arising from this treatment is its preservative effect by the exclusion of moisture.* The friction here mentioned is also of great importance from its stiffening effect, rendering the arch, when sufficient, equivalent to one of whole timber; it primarily depends upon the compression induced by the screw-bolts, straps, etc., that act upon the arch in the direction of its radius; these, therefore, should be numerous and well tightened up. When brace-sockets are employed, the bolts connect-

* Or, the planks being dressed with the plane, white-lead paint may be used, as in building large masts.

ing them with the arch contribute to this end, and, of course, effect a corresponding saving; these bolts, with the addition of screw-bolts, such as shown in Figures 138 to 140, would constitute a good arrangement. Trenailing forms a cheap additional means of stiffening the arch, but renders the removal of a plank, in the event of its becoming decayed, an almost impossible operation.

Figures 141 and 142 show an American method of drawing the planks together; by the use of these dovetail-gibs and wedges and trenails, a stout arch may be formed without the aid of iron.

For the case of Figure 38, the arch should have a broad section, and then it might be expedient to use more than one system of braces; a section of such an arrangement as is meant is shown by Figure 144 (see also foot-note, page 35).

THE SPRINGING of a *tied* arch is a part that demands great attention. The most perfect arrangement is that which distributes the thrust of the arch equally over the section of the tie. Hence the method of Figure 146 is perhaps the best for an arch and tie of the nature shown; each of the iron steps is fixed to its plank of the tie by numerous screw-nails. Figures 148 and 149 are methods for the case of a laminated arch with a tie beam of a broad and thin section.

IRON STRUCTURES.

The lower beam should be constructed of wrought iron; the upper one may be of either kind, and, generally speaking, the tubular will be the best form that can be given to it, for which form wrought iron, as a material, is perhaps the more preferable; but is the more difficult of the two to manage when the member is of the arched form, as the whole must be put together on the ground and thence raised with powerful tackle, or the riveting must be performed, with great inconvenience, on the centring.

The springing of a tied wrought-iron arch may be formed, as shown in Figure 150, by surrounding the end with strong

angle-iron, which is then riveted to the flat end of the tie; or by inserting the foot of the arch into a cast-iron socket, as is done with the wooden one in Figure 148. A cast-iron arch may have its terminal pieces formed, as in Figure 151, with solid expanded soles, bolted down to the tie.

For untied arches a common cast-iron socket, with its bearing against the abutment expanded and normal to the arch, will generally be sufficient.

CHAPTER IV.

SUPPLEMENTARY.

It is not here intended to institute a comparison of the merits of the various designs, as that would demand a very large space, depending so much, as the excellence of a structure does, upon its adaptation to the particular combination of the requirements and circumstances of situation in which it may be assumed to be placed; such an extended enquiry would be more appropriate to a treatise on bridges. But we shall recapitulate a little, and add anything that may suggest itself as closely connected with the subject of the work, but which came not conveniently under any of the foregoing heads.

Though the first class may be surpassed in strength by those wherein the arch is used, it possesses many advantages in other respects; foremost of these is its simplicity, offering easy construction; there is great uniformity of parts, which, especially when cast iron is employed, may be a source of economy; it admits of a roadway at either level, without interfering with the horizontal bracings, of which only two series are necessary; it exerts no horizontal thrust; and, as practised by Mr. Stephen-

son, with the tubular bridges, it may be put together at some convenient spot, and thence removed in a nearly complete state to its permanent site; or, if erected there, only a very simple and inexpensive scaffolding will be necessary.

The form, of which Figure 152 is a transverse section, may be considered as a variety of the first class; in it the main braces require to be longer; the horizontal braces are also necessarily longer, but this renders them more powerful; its advantages are, that by concentrating the upper beams into one, their joint strength against compression will be increased; that one series of horizontal braces is sufficient; and that it may be roofed in at very little expense.

Speaking generally, the internally-braced arches are superior in strength to those with external bracing; the advantage attends the deepening of the bracing towards the mid-span.

When a structure of class fourth has the arches parallel, it may possess the recommendation, when cast iron is employed for the bracing, of uniformity of the castings.

The method of connecting the braces to the main beams by means of trenailing may appear very rude, but it is capable of producing considerable strength; and, as it may in some situations be a convenient one, the reader is referred, if unacquainted with better data, to a table of the strengths of trenails, given in the last edition of the "Encyclopædia Britannica," art. "Shipbuilding."

We are not aware of any satisfactory experiments having been made to determine that strength of timber which arises from the resistance to severance by the sliding longitudinally of a part of the material, upon which primarily depends the strength of the socketed wooden braces against a tensive strain. This strength or resistance to severance will be increased in the dovetail form by the lateral compression, but it may be lessened in the square-headed variety, if care be not taken to obviate any splitting action—that is, any force acting so as to wrench the fibres separate.

The following observations are made in order to show the applicability of the formulæ to the forms of bracing seen in Figures 15, 16 or 17, and 18: In Figure 15, if a weight be placed at c , the portion of it which must be imposed on the pier to the left side will not influence cb or ba , as ba cannot act the part of a tie. It is therefore conveyed by cd , da , etc.; and through these parts would, in like manner, pass the strains arising from all the weights placed to the right of c , which would ultimately arrive at the pier to the left side. When all such weights are alone on, the strains in cd and da are maxima. A similar course would be pursued by the strains in the bracing of Figure 21. The effect of the portion of the weight at c , which is to be supported by the pier to the right, must flow through the strut, and not through cb . Thus it is readily seen by what loading the strain in a part is rendered a maximum, and also the formula by which its amount will be given, after substituting the proper values for the symbols.

The bracing must be continued throughout the length of the structure. This will be clearly seen by inspecting Figures 153 and 154, which show changes that may take place when only a part is braced.

An arch may be employed without having the bracing arranged along its intrados or extrados; it may be stiffened by being connected with a braced structure of class first. Figures 82, 83, 84, 155, and 156 are illustrative of this compound arrangement. In the arch, the radius of curvature may be made to diminish more rapidly in approaching the crown than if the curve were a parabola, as it is principally required to aid the class-first portion at the mid-span. The maximum strains in the parts of the class-first portion will be very much reduced by the addition of the arch. Figure 156 is a good design for very large structures, as it permits of the roadway being at the lower level, and as then the horizontal bracing between the upper beams would be sufficiently high; the arch and each of the straight longitudinals will require horizontal bracing.

In class first a saving of half the scantling at the centre of

the main beams might be effected, if a pressure of *exactly* $\frac{R.W}{16D}$ were made to act tensively at each end of the upper beam, and compressively at the ends of the lower one; for then one-half of the load would be borne up on the principle of the girder supported at both ends, while the other half would be upheld on the principle of a projecting beam fixed at one end and unsupported at the other. The strains in the longitudinals would be subject to a peculiar arrangement. Their maxima would occur at the mid-span and at the extremities, and would be $= \frac{R.W}{16D}$, or half what would otherwise take place at the centres, and the strains at the extremities would be tensive should that at the centre be compressive, and *vice versa*; and at the points midway between the mid-span and the extremities, where the strains change their character, the longitudinals would of course be free from all action. Advantage is taken, to some extent, of this compound action in the Britannia Bridge, by the method pursued in uniting the tubes over the central pier or tower.

The following observations are an extension of a principle employed in the latter part of Chapter V., Part First. Let Figure 64½ represent a portion of a voussoired arch; now, if the line of pressures, from not being capable of conforming to the curve of the arch, rise above it at *a*, as shown by the dotted line, the joint *a b* will open at *b*. But let us suppose the joint at *b* to become solid, or that it is made good there by means of bolting, the line may then rise above *a* or any joint sufficiently united at its lower point; and might also sink below a part of the arch, were the upper points of the joints tied. This is the transition condition between an arch and a girder action; it is that in which a cast-iron (or other continuous rigid) arch may be placed by an uneven loading too severe for its depth of section. When strengthened by bracing, the joints cannot open, as the voussoirs are then prevented changing their proper radial directions.

The strains in the several parts of Figure 31 will be exactly the same in amount, but of opposite character, with those in

the corresponding parts of Figure 38. And generally, whatever has been said with regard to the comportment of the line of *pressures* in the braced erect arch applies, in a reversed state, to the curve or *line of tensions* in the braced inverted arch, or rigid suspension bridge.

And here the writer would almost be induced to speculate upon the possible extent to which bracing might be employed, imagining structures wherein each brace would offer in itself no contemptible example of braced construction; but he restrains himself, deeming the opportunity objectionable for such suggestions.



APPENDIX.

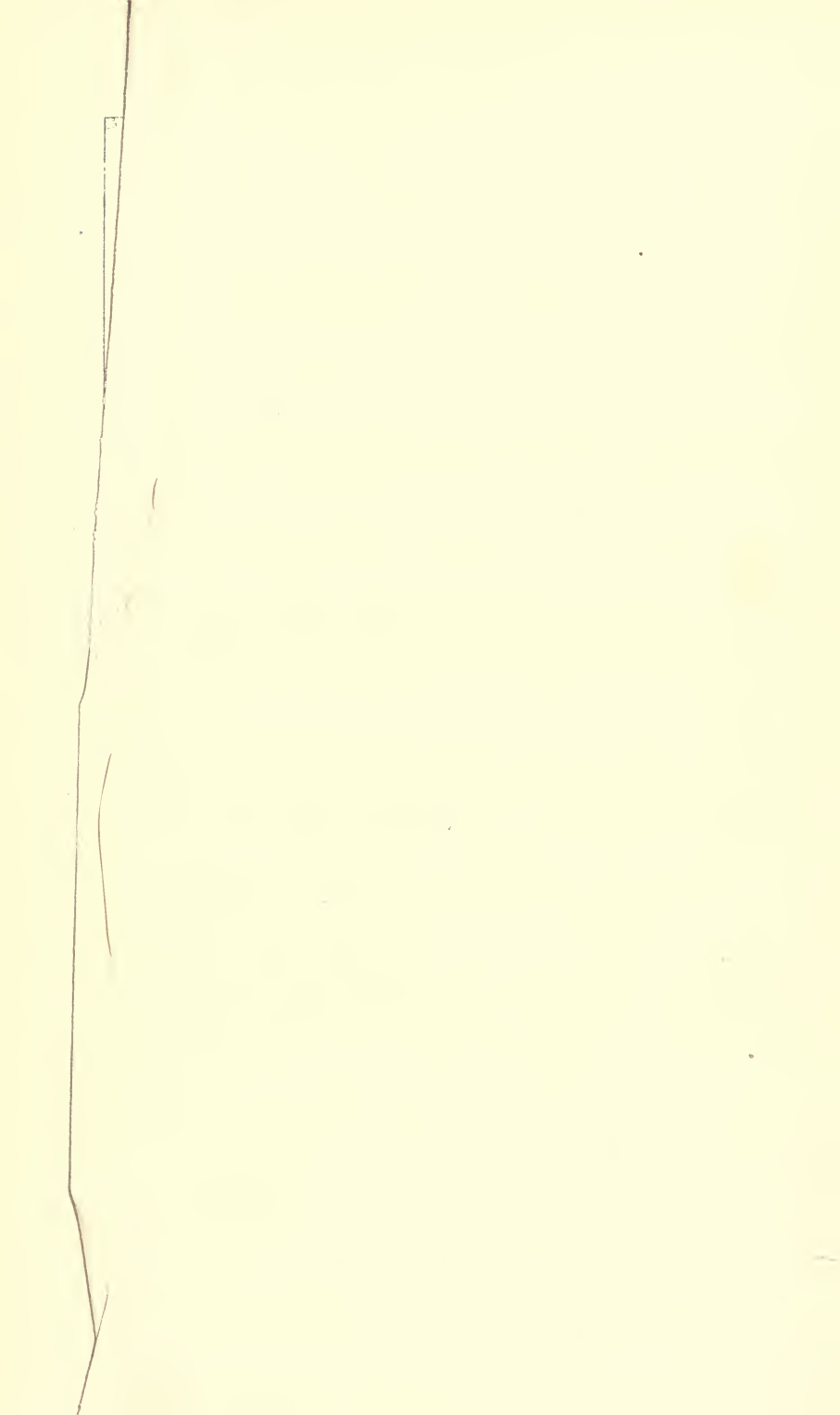
WHEN the printer had made considerable progress, and consequently when it would have been very inconvenient to notice it at the proper places, I observed, at page 391 of the *Civil Engineer and Architects' Journal* for December, 1850, an account of the failure of a girder bridge in which the triangular method of bracing was employed. I was not previously aware of a patent having been granted for any structure employing the method, however imperfectly.

The instance of failure of the structure of which I speak offers an opportunity of employing the results obtained in Chapter III., Part First. I will, however, merely outline the application, as it is a very simple matter: The *braces*, on approaching the piers, ought to increase in strength, as seen in Figure 47. In that figure, 1 *m* will represent the fractured brace; it had to undergo a very great *tensive* strain, but was made of *cast iron* of no greater section than the central braces; it—and we cannot wonder at it—broke. The great virtual loading impressed on the point 2 by the brace *l* 2 being thus left to be upheld by the transverse strength of the upper longitudinal part, that part became fractured near the said point. The remaining fracture probably resulted from the direct action of the load on the portion left projecting. It is not stated that any further breakage took place; if none did, the circumstance may be explained by the peculiar construction of the “girder.” The lower longitudinal part, which was composed of wrought-iron chain-work, though it could not give aid in supporting the point *m*, while it continued quite horizontal, offered, after the failure, sufficient to uphold the wreck;

and the terminal braces of what remained as a "girder" proved sufficient for the diminished load and span.

The writer of the article suggests that the accident would have been averted, if the lower longitudinal part had been of cast iron, and all cast in one piece. The longitudinals require very great strength at the mid-span; consequently, if of cast iron, the lower one would demand a very large section there to resist the great tensive strain. But let it not be supposed that I approve of the mixture of cast and wrought iron as managed here. The danger of the combination lies in this: that the elasticity of the wrought-iron tie may be so great as to allow the points (say j and k , Figure 47) of the braced elements to separate further than the elasticity of the upper cast-iron longitudinal can permit without fracture (at point 4). If, however, the upper longitudinal member be jointed at each of the points, 2, 3, 4, 5, etc., the latter danger will be provided against.











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